Third Quarterly Report: Components of a Sediment Transport Model for San Francisco Bay

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May 7, 1993

Introduction

The purpose of this report is to present the primary components of the sediment transport model currently under development at the U.S. Geological Survey (USGS). The main features of the model are described, followed by a discussion of several important assumptions and limitations inherent to the model. Following the model description, some sample output is presented and discussed. The report continues with a summary of the goals for the fourth and final quarter of this project year and concludes with recommendations for further research.

Sediment Transport Model

The sediment transport model being developed at the USGS consists of the following components:

1) A two-dimensional depth-averaged finite difference model is used to describe the hydrodynamics for San Francisco Bay. This model, TRIM, (Tidal Residual Intertidal Mudflat) has been thoroughly tested and validated against field measurements in the bay and is described in detail elsewhere (see, for example, Cheng et al. (1993)). TRIM includes a routine that solves the advection-diffusion equation and may be easily adapted for computing the transport of sediment given proper formulations of the source/sink terms in this equation.

2) A submodel has been developed which describes the erosion and deposition of cohesionless sediments. Results of this submodel are input into the advection-diffusion equation as source/sink terms.

3) The erosion and deposition of cohesive sediments are computed by another submodel. Results of this submodel are incorporated into the advection-diffusion equation as source/sink terms.

4) A layered bed model is used to represent the sediment bed. The model accounts for depthwise variations in density and shear strength of bed material below the sediment/water interface.

5) Finally, the increase in effective shear stress at the sediment/water interface due to the interaction of waves and currents in shallow regions of the bay is determined by another model component.

The details of each of these components are described in the following sections.

Hydrodynamic Model

As stated earlier, the formulation of the hydrodynamic model, TRIM, is described elsewhere and, therefore, is not presented in detail here. However, output from the model and portions of the formulation particularly relevant to computing the sediment transport are discussed below.

Output from TRIM includes depth-averaged velocities and water surface elevations at any point within the computational domain. Computed values of the velocities may then be used in the following advection-diffusion equation for any desired non-reactive constituent, C:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y} = \frac{1}{H}K_h\left[\frac{\partial}{\partial x}(H\frac{\partial C}{\partial x}) + \frac{\partial}{\partial y}(H\frac{\partial C}{\partial y})\right] + \alpha C + \beta \qquad (1)$$

where

t is time,
(U,V) are the depth averaged velocity components in the (x,y) directions,
H(x,y) is the total water depth,
K_h is the horizontal diffusivity coefficient,
α is the coefficient of the concentration-dependent source/sink term, and
β is the concentration-independent source/sink term.

Equation 1 is used, following proper determination of the source/sink terms, to compute the depth-averaged concentrations of suspended sediment within the computational domain.

Cohesionless Sediment Submodel

As discussed in the previous quarterly report ("Summary of Literature Relevant to Sediment Transport in San Francisco Bay"), the sources and sinks of cohesionless sediments are typically evaluated by first computing an "equilibrium" transport value for the given flow conditions and then applying enough erosion or deposition such that the suspended sediment concentration approaches the computed equilibrium value. Based on reviews of White et al. (1975, 1976), Heathershaw (1981) and others, two formulations have been selected for determining the equilibrium sediment concentration in the sediment transport model. The first expression is that given by Engelund and Hansen (1967):

$$q_{st} = 0.05\rho_s\sqrt{U^2+V^2}\left[\frac{d_{50}}{g}(\frac{\rho_s}{\rho}-1)\right]^{1/2}\left[\frac{\tau_b}{(\rho_s-\rho)gd_{50}}\right]^{3/2} \quad (2)$$

where

q_{st} is the equilibrium sediment transport rate,
ρ_s and ρ are densities of the sediment and water, respectively,
d_{50} is the median grain size,
g is the gravitational acceleration, and
τ_b is the bottom shear stress.

The second formulation which may be used to compute the equilibrium transport is that presented by Ackers and White (1973):

$$G_p = C(\frac{F_{gr}}{A}-1)^m\frac{sd_{35}}{H}(\frac{\rho}{\tau}\sqrt{U^2+V^2})^n \quad (3)$$

where G_p is the equilibrium sediment transport rate, the mobility number, F_{gr}, is defined as

$$F_{gr} = \left[\frac{\tau^n\tau'^{(1-n)}}{\rho g d_{35}(s-1)}\right]^{1/2} \quad (4)$$

and and d

d_{35} is the sediment grain size for which 65% by weight is coarser,
s is the specific gravity of the sediment,
τ is the total boundary shear stress (including effects of bedforms and geometry),
n is a coefficient which expresses the relative importance of bedload and suspended load transport, and
τ' = boundary shear stress due to particle roughness only (stress that would be exerted on a plane bed for given flow conditions).

The coefficients n, A, C, and m are given by the following expressions:

For $1 < D_{gr} \leq 60$:

$$n = 1 - 0.56\log D_{gr}$$

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14$$

$$\log C = 2.86 \log D_{gr} - (log D_{gr})^2 - 3.53$$

$$m = \frac{9.66}{D_{gr}} + 1.34.$$

For $D_{gr} > 60$:

$$n = 0$$
$$A = 0.17$$
$$C = 0.025$$
$$m = 1.5.$$

The parameter D_{gr} is a dimensionless grain size defined by:

$$D_{gr} = d_{35} \left[\frac{g(s-1)}{\nu^2} \right]^{1/3} \tag{5}$$

where ν is the kinematic viscosity of the fluid.

The user has the option of choosing either of the above formulations to compute the equilibrium sediment concentration. In addition, both submodels may be used in order to compare the results of the two methods.

The depth-averaged erosion and deposition rates are computed using the following expression given by Falconer and Owens (1990):

$$E = \gamma V_s (C_e - C) \tag{6}$$

where E is the net rate of erosion or deposition per unit area of bed, C_e is the depth-averaged equilibrium concentration determined from one of the two methods described above, V_s is the particle settling velocity, and γ is a profile factor defined as the ratio of the bed concentration, C_0, to the depth-averaged concentration, C. Ideally, γ should be estimated from field measurements of sediment concentration profiles. For their application (Humber Estuary, England), Falconer and Owens (1990) found that the optimum value of γ was approximately 1.0.

Cohesive Sediment Submodel

The equations used to describe deposition and erosion of cohesive sediments are very similar to those used in the U.S. Army Corps of Engineers sediment transport model, STUDH (See Hauck et al. (1990)). The deposition rate is given by Krone's (1962) equation:

$$S = \begin{cases} -\frac{2V_s}{H} C (1 - \frac{\tau_b}{\tau_d}) & \text{for } C < C_c \\ -\frac{2V_s}{H C_c^{4/3}} C^{5/3} (1 - \frac{\tau_b}{\tau_d}) & \text{for } C > C_c \end{cases} \tag{7}$$

where

S is the deposition rate,
V_s is the fall velocity of a sediment particle,
C is the mean sediment concentration in the water column,
τ_b is the bed shear stress,
τ_d is the critical shear stress for deposition, and
C_c is the critical concentration (= 300 mg/l for San Francisco Bay, as determined by Krone (1962)).

The expression for particle erosion is computed from Ariathurai's (Ariathurai et al. (1977)) adaption of Partheniades' (1962) findings:

$$S = \frac{P}{h}\left(\frac{\tau_b}{\tau_e} - 1\right) \quad \text{for } \tau_b > \tau_e \tag{8}$$

where P is an erosion rate constant, τ_e is the critical shear stress for erosion, and S is the rate of sediment erosion.

Finally, erosion due to mass failure of a sediment layer is given by:

$$S = \frac{T_L \rho_L}{h \Delta t} \quad \text{for } \tau_b > \tau_s \tag{9}$$

where

T_L is the thickness of the failed layer,
ρ_L is the density of the failed layer,
Δt is the time interval over which failure occurs, and
τ_s is the bulk shear strength of the layer.

Estimates of the empirical coeffecients in these equations are based on flume studies (Krone, 1962) and limited field measurements (e.g. Hauck et al., 1990).

Bed Submodel

The bed model consists of a series of "n" layers, each of which may have a different thickness, dry density, and shear strength. Following the model described by Hayter (1983), a near-bed layer of very high concentration suspension is included. This layer has a significantly lower shear strength than the underlying consolidated layers and is allowed to erode by mass failure only, as opposed to particle by particle erosion. The inclusion of this layer is justified by experimental and field observations which suggest that during the deposition process a very high-concentration "stationary" suspension forms at the top of the sediment bed. This stationary suspension has a small but finite shear strength, and at shear stresses above the critical value is eroded in mass and returned to suspension within the water column. Below the stationary suspension layer is a series of consolidated layers which generally erode particle by particle as described by Equation 8. If the stationary suspension layer is not eroded within a specified consolidation period, T_c, "consolidation" of this layer occurs. During "consolidation" the sediment mass contained in the stationary suspension is transferred to the underlying consolidated layer and converted to the appropriate dry density and shear strength of the underlying layer. Following this consolidation process no new stationary suspension layer forms until the next period of deposition.

Bottom Shear Stress

The expression used to compute the bottom shear stress in TRIM is given by a Manning-Chezy formulation, i.e.

$$\frac{\tau_x^b}{\rho_0} = \frac{g\sqrt{U^2 + V^2}\, U}{C_z^2} \tag{10}$$

and

$$\frac{\tau_y^b}{\rho_0} = \frac{g\sqrt{U^2 + V^2}\, V}{C_z^2} \tag{11}$$

where τ_x^b and τ_y^b are the bottom shear stresses in the (x,y) directions, U and V are the horizontal velocities in the (x,y) directions, ρ_0 is a reference density, and the Chezy coefficient, C_z, is related to Manning's n (in metric units) by:

$$C_z = \frac{H^{1/6}}{n} \tag{12}$$

where H is the total depth of the water column.

Interaction between the turbulent wave and current boundary layers in shallow areas can significantly increase the effective bottom shear stress felt by the bottom sediment. To account for this effect, the sediment tranport model includes a sub-model developed by Madsen and Wikramanayake (1991). Their model includes a modified representation of the eddy viscosity, allowing it to be continuous at the top of the wave boundary layer and to vary with time. The shear stress due to current alone is assumed to be known and is computed using Equation 12. Madsen and Wikramanayake (1991) show that their formulation agrees well with experimental data and have simplified the computations so that they can be solved quite efficiently. Predictions of wave characteristics may be made based on actual field measurements of wave parameters or by methods described in U.S. Army Corps of Engineers (1984).

Summary of Model Formulation

The components of the sediment transport model described in the preceding sections are combined into one comprehensive finite difference model in which the erosion and deposition of sediment is decoupled from the hydrodynamics. Thus, changes in bathymetry resulting from erosion and deposition are assumed to be negligible and are not incorporated into the hydrodynamic model. At each time step, the hydrodynamic properties are computed and are used as input to the routines which compute erosion, deposition, and transport of sediment. Cohesionless and cohesive sediments do not interact and are treated independently in the model. The sediment type can be entirely cohesionless or entirely cohesive by appropriate specification of the initial and boundary conditions. Presently only one characteristic grain size may be specified for the cohesionless sediments.

At the open boundaries of the computational domain, input to TRIM requires specification of water surface elevation and salinity. The sediment model additionally requires specification of suspended sediment concentrations (of both cohesive and cohesionless sediments) at these boundaries.

Accuracy of the results produced by this model are limited primarily by the lack of basic understanding relating to processes governing erosion and deposition, particularly with respect to cohesive sediments. The empirical expressions used to describe these processes in the model were developed based on observations from experiments performed in the laboratory and may vary significantly from true behavior in the field. Unfortunately, field measurements of erosional and depositional sediment properties are extremely difficult to obtain and, therefore, are quite scarce.

The present sediment transport model is also limited by the two-dimensional depth-averaged formulation. During periods of high freshwater inflows into the bay, significant vertical variations in salinity can develop leading to very different hydrodynamic behavior than that which occurs under relatively uniform depthwise conditions. For conditions where depthwise variation becomes significant, a three-dimensional model of the hydrodynamics would be most appropriate.

Sample Model Output

The sediment tranport model is currently running under hypothetical conditions, and the sensitivity of the model is being investigated. Variables output from the sediment transport model include velocity, salinity, water surface elevation (from the hydrodynamic model), concentration of suspended sediment (both cohesionless and cohesive), and net erosion or deposition of the bed. The model results may be examined by means of computer graphics time sequences (computer graphic animations) of a particular variable or as time series plots of variables at selected grid points. As an example of typical output, Figure 1 shows the spatial distribution of areas of erosion and deposition from a trial model run. Figure 2 shows time series plots of speed, water surface elevation, and concentration of cohesive sediment at a selected point. Note that these results are only presented as examples of typical output and have not yet been validated or verified by field data.

Goals for the Coming Quarter

The primary goal of the fourth quarter is to continue refinement of the sediment transport model while placing particular emphasis on evaluating the model sensitivity. This evaluation is necessary in order to determine the relative importance of the various input parameters. In addition, the USGS sediment transport model will be tested against available sediment data from San Francisco Bay. If possible, preliminary applications of the model will be carried out in regions of the bay which are of particular interest to this project.

Recommendations for Future Work

Although a great deal of progress has been made towards development of a sediment tranport model, evaluation of the model capabilities and results will have only just begun by the end of the first project year. Recommendations for future development of the model over the next 1-2 years include the following items:

1) Continued analysis of the sensitivity of relevant parameters and preliminary model applications should be performed in order to further evaluate the capabilities and limitations of the model.

2) Based on results of preliminary model runs and review of the relevant literature, recommendations for appropriate laboratory and field measurements most useful for further improvement of the model parameterizations should be developed. As stated earlier, scarcity of field and laboratory sediment data and lack of a basic understanding of erosional and depositional processes are the primary factors limiting the accuracy of the model. Further progress in the development of the model is strongly dependent on obtaining more reliable field and laboratory data.

3) Extensive testing of the sediment tranport model against any recent field data acquired in San Francisco Bay should be carried out. In addition, the model's usefulness in addressing issues related to the disposal and redistribution of dredged material should be thoroughly evaluated.

4) Extension of the two-dimensional sediment transport model to three dimensions may be considered. However, this phase of the research depends on the results of a three-dimensional hydrodynamic model, which is currently under development.

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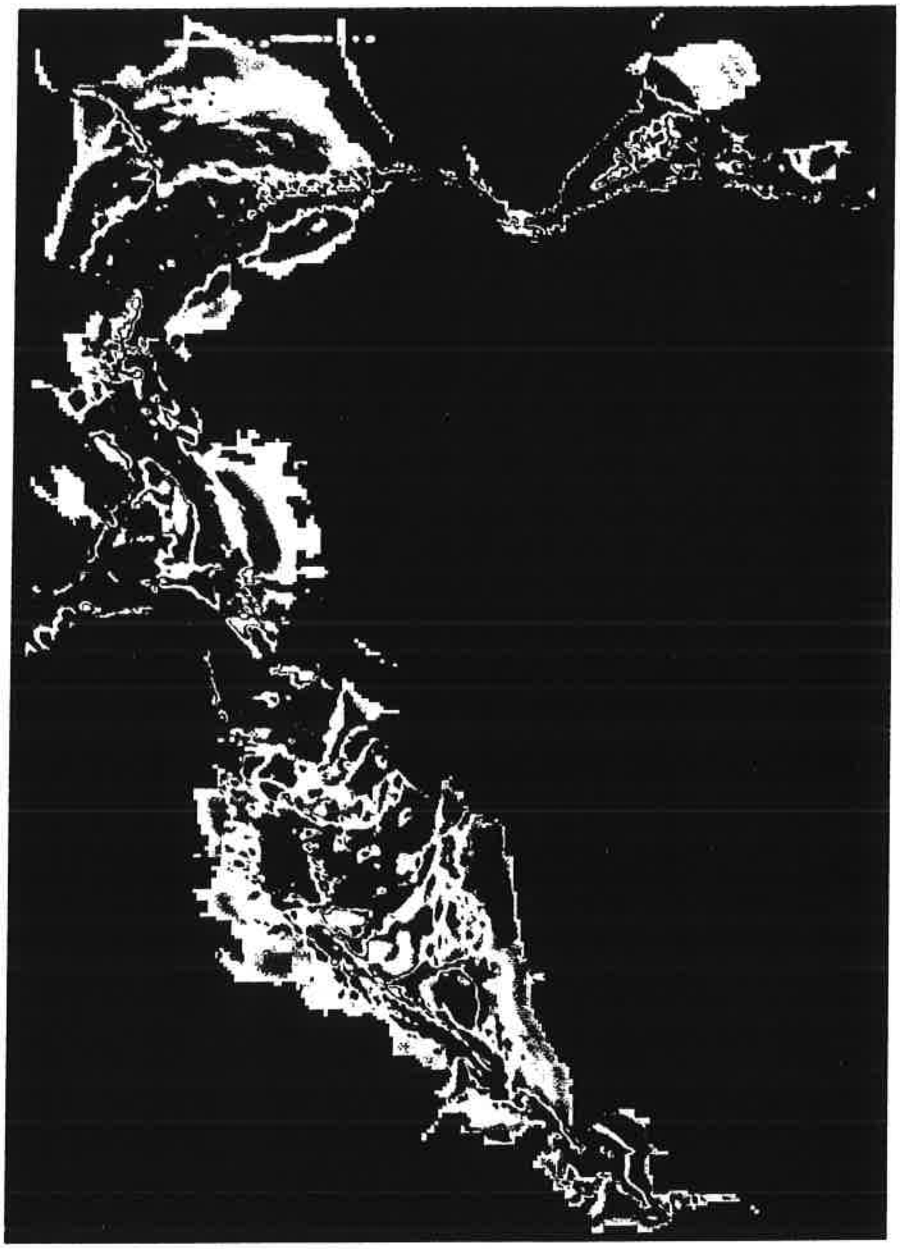

Figure 1: Sample spatial distribution of regions of erosion and deposition in San Francisco Bay. Warm colors (reds) indicate areas of deposition and cool colors (blues and greens) indicate areas of erosion. Yellow indicates little or no net erosion or deposition.